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A FEEDBACK MECHANISM IN ANNUAL RAINFALL, CENTRAL SUDAN

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ABSTRACT

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Annual rainfall in many parts of the world is an independent process. Yet annual rainfall series in some regions of Africa show characteristics incompatible with such an hypothesis.

The annual rainfall process in Central Sudan is weakly dependent. The hypothesis that the dependence is due to a "feedback mechanism" is investigated using a mathematical model based on the water balance of the neighbouring region, Bahr Elghazal.

INTRODUCTION

The location of the region of Central Sudan and the rainfall gauging stations therein, are shown in Fig. (1). An analysis of the annual rainfall series at these stations indicates a significant degree of dependence. The autocorrelation coefficient at lag one was calculated as 0.34 at Kosti, 0.29 at Obeid and 0.19 at Nihood. Since rainfall records from individial stations are more noisy than regionally averaged records, the regional average is likely to have a higher degree of dependence but because the station-density of the region is very poor it is not useful to calculate this quantity.

Dependence in annual rainfall series implies that the amount of rainfall in any one year depends to some extent on the amount in the previous year or years. Feedback mechanisms are the physical mechanisms which explain how this can occur.

Annual rainfall in many parts of the world is nearly an independent process. Yevjevich (1964) concluded that scientists in hydrology, meteorology and other connected disciplines should adjust their attitudes and philosophies to the fact that annual precipitation at a point or over an area is nearly an independent process. He mentioned that weak indications of dependence, observed in annual rainfall, may be explained by inhomogeneity or inconsistency in the data, introducing nonstationarity and hence a pseudodependence.

It was thought necessary, before studying possible feedback mechanisms to investigate the series carefully and to confirm that dependence is a genuine

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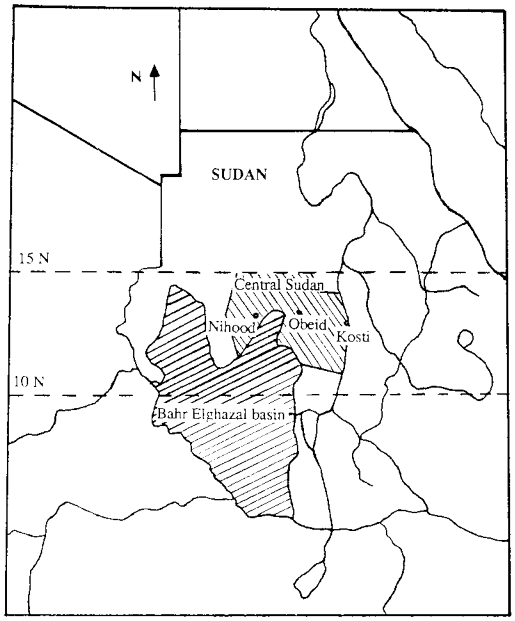

Fig. 1. Regions of Central Sudan and Bahr Elghazal.

feature of the annual rainfall process in the region. Later sections of this paper review the relevant feedback mechanisms, previously suggested for explaining dependence, and the paper concludes with an hypothetical explanation of the observed dependence in annual rainfall in the region of Central Sudan.

DEPENDENCE IN ANNUAL RAINFALL SERIES

The objective was to confirm that the observed dependence in the annual rainfall series in the region, was not due to non-stationarity of the series. The procedure was to use the Kendall's rank statistic in testing the null hypothesis that the series was a sample from a population which is stationary in the mean. For the series which were accepted to be stationary, the autocorrelation coefficient at lag one was calculated and tested. Anderson's test was applied with the null hypothesis that the series was a sample from an independent population, against the alternative one sided hypothesis that the series was a sample from a positively dependent population. It was considered that a sample size of 40 was sufficient for estimating the autocorrelation coefficient at lag one.

Kendall rank statistic is a measure of the correspondence or the correlation between any two rankings of a series. In applying Kendall test for testing the stationarity in the mean, one of the two rankings should be the historically observed sequence, the other ranking can be the series sorted in an ascending or descending order, usually it is taken as the rising series. Hence a value of the statistic close to a negative one indicates a falling trend. For a large sample from a stationary population the value of the Kendall rank statistic asymptotically approaches zore.

The procedure was applied to samples taken from different parts of the annual rainfall series for the three stations in the region. Firstly, samples of size 40 were taken beginning every ten years. Secondly the series was split into two samples of equal size. Lastly the sample with approximately zero Kendall statistic was located and tested. Table 1 shows the results of these tests.

From the results at the three stations it was confirmed that the annual

TABLE 1

Results of the stationarity and independence tests

Station	Period of sample	Kendall statistic	Stationarity test result	Autocorrelation coefficient at lag one	Anderson's test result
Obeid	1902–1942	0.06	A*	0.39	R′
	1912–1952	0.23	R		
	1922–1962	0.12	A	0.14	A′
	1932–1972	−0.25	R		
	1942–1982	−0.28	R		
	1902–1943	0.08	A	0.38	R′
	1944–1986	−0.30	R		
	1902–1972	0.01	A	0.29	R′
	1902–1986 (full record)	−0.09	A	0.29	R′
Kosti	1909–1949	0.05	A	0.38	R′
	1919–1959	−0.05	A	0.41	R′
	1929–1969	−0.08	A	0.35	R′
	1939–1979	0.04	A	0.35	R′
	1909–1946	0.08	A	0.32	R′
	1947–1986	−0.16	A	0.42	R′
	1909–1975	0.01	A	0.33	R′
	1909–1986 (full record)	−0.10	A	0.34	R′
Nihood	1912–1952	0.15	A	0.27	A′
	1922–1962	−0.05	A	0.16	A′
	1932–1972	−0.17	A	0.17	A′
	1942–1982	−0.22	R		
	1912–1947	0.21	A	0.27	A′
	1948–1986	−0.27	R		
	1912–1972	0.00	A	0.18	A′
	1912–1986 (full record)	0.13	A	0.19	A′

*A = Accept the null hypothesis that the series is stationary (i.e. this hypothesis was not rejected at the 5% level of significance) R = The stationarity hypothesis was rejected at the 5% level of significance; A′ = Accept the null hypothesis that the series, which has already been considered stationary, is from a serially independent population (i.e. this hypothesis was not rejected at the 5% level of significance); R′ = The null hypothesis, of independence, was rejected at the 5% level of significance, the alternative hypothesis of positive dependence, was accepted.

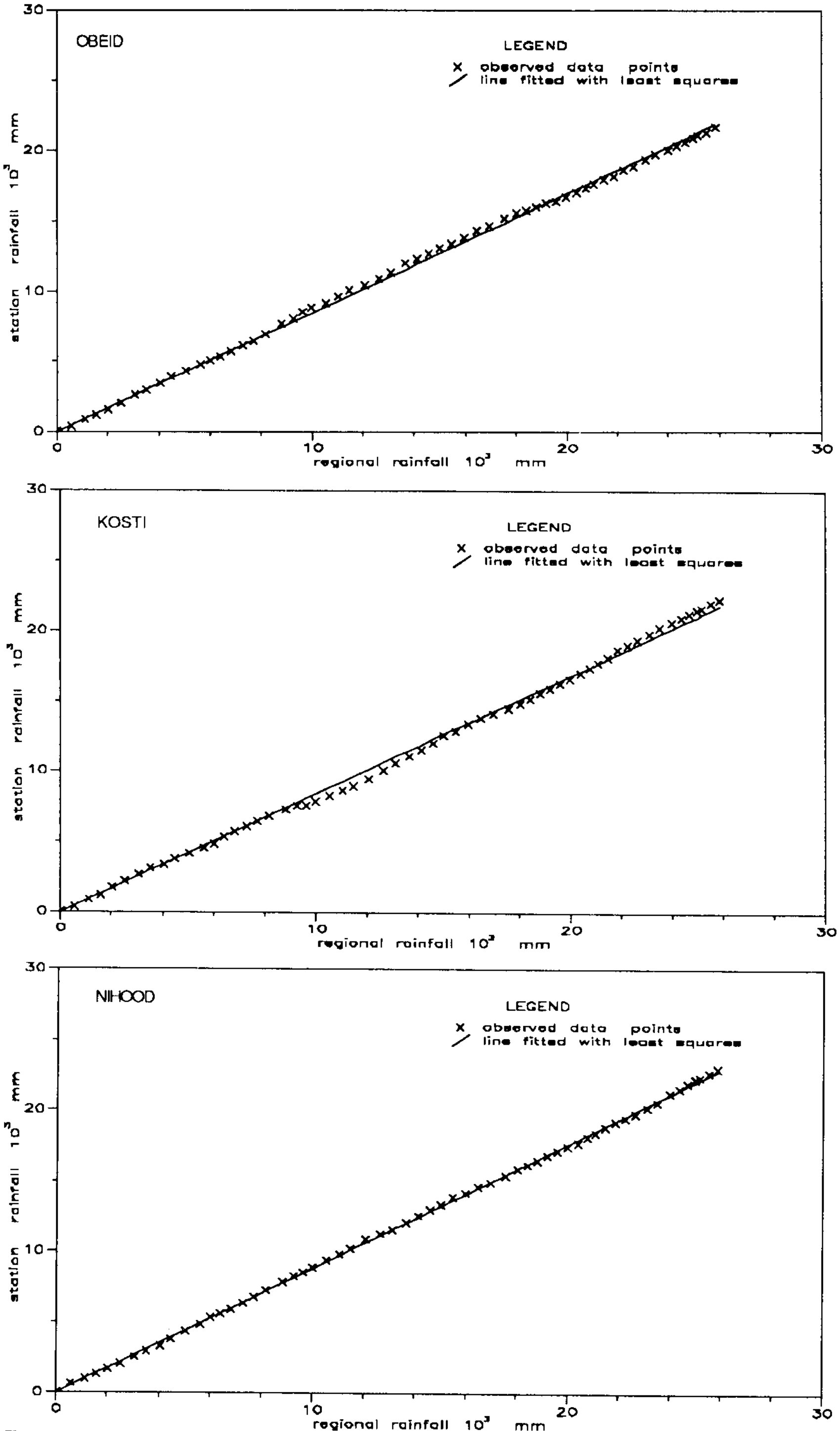

Fig. 2. Double-mass analysis.

rainfall series in the region is positively dependent. Nonhomogeneity or inconsistency in the data may have helped to magnify the observed dependence.

The homogeneity of the records at the three stations was investigated by double-mass analysis. The results aer shown in Fig. (2). From the double-mass curves it is clear that the records at the three stations are reasonably homogeneous.

The author investigated the annual rainfall series in the region and concluded that the first order autoregressive model fits the series fairly well (Eltahir, 1987).

FEEDBACK MECHANISMS IN ANNUAL RAINFALL

A UNESCO–WMO report (1985) summarises some of the feedback mechanisms suggested for explaining some characteristics of drought. The following is an account of two of these.

(1) A decrease in rainfall or overgrazing results in loss of vegetational cover and an increase in albedo. The latter results in local loss of heat and a consequent lateral temperature gradient which induces a circulation such as to restore equilibrium with warmer surroundings.

In the Sahel region of Africa such a mechanism results in a circulation opposite to the one responsible for the rainfall and hence a shift of the intertropical convergency zone (ITCZ) to the south resulting in less rainfall. This mechanism is known as Charney's hypothesis.

The Charney mechanism is relevant to Central Sudan and possibly to the northern boundaries of the region. The problem of overgrazing which feeds the mechanism, is a real problem in the region.

(2) In many parts of the world an important source of moisture to supply rainfall is the evaporation of soil moisture from neighbouring areas. Since the soil moisture itself derives from previous rainfall, we might expect wet periods to persist longer than what chance alone would indicate and dry periods even longer, especially in areas were soil moisture is almost the only moisture routinely available. This mechanism is similar to the one suggested in the next section.

A FEEDBACK MECHANISM IN ANNUAL RAINFALL, CENTRAL SUDAN

Although the degree of dependence in annual rainfall in Central Sudan is significant, it is very low. The dependence structure of the series accounts for less than 10% of the variance around the mean. Hence the mechanism which is responsible for this dependence is a weak one.

A feedback mechanism is suggested for explaining dependence in annual rainfall in the region of Central Sudan, in which the carry over factor is the water which precipitated in the neighbouring southern region, the Bahr Elghazal basin.

All the precipitation in this basin evaporates, either in the same year or in the following years, and contributes to precipitation in Central Sudan. A high level of rainfall in Bahr Elghazal in any year thus results in increased evaporation in the following years and, hence, higher levels of rainfall in those years in Central Sudan. But the annual rainfall series in Bahr Elghazal is similar to the annual rainfall series in Central Sudan. Hence high level of rainfall in the entire region in any year will probably result in high levels of rainfall in Central Sudan in the following years and vice versa.

Before developing the equations of this model, the geography and location of Central Sudan and the water balance studies of Bahr Elghazal basin are reviewed.

GEOGRAPHY OF CENTRAL SUDAN

The geographical location of the Central Sudan is shown in Fig. (1). It is an arid region with mean annual rainfall ranging between 300 and 400 mm. The length of the rainy season is about six months. The region is bounded on the north by the sub-Sahara region of Africa which is very dry with annual rainfall less than 100 mm. It is bounded on the west by northern Darfur which is also a dry region with annual rainfall between 100 and 300 mm., and on the east by the Gezira region of Sudan. It is bounded on the south by Bahr Elghazal basin, a region of, normally, high rainfall levels, with a rainy season of eight months. Almost all the clouds which precipitate in Central Sudan pass over this basin. The area of this basin is half a million square kilometers, equivalent to that of France.

WATER BALANCE OF THE BAHR ELGHAZAL BASIN

The water balance of this basin is particularly simple. Two processes dominate the hydrology, namely rainfall and evaporation, while all the other processes are negligible. An important hydrological feature in Bahr Elghazal basin is the central swamp with an area ranging between about 84,000 km^2 at the end of the rainy season to about 16,000 km^2 at the end of the dry season.

Dekker (1972), who conducted a water balance for the Nile basin pointed out that no water from the Bahr Elghazal system, appears to reach the White Nile.

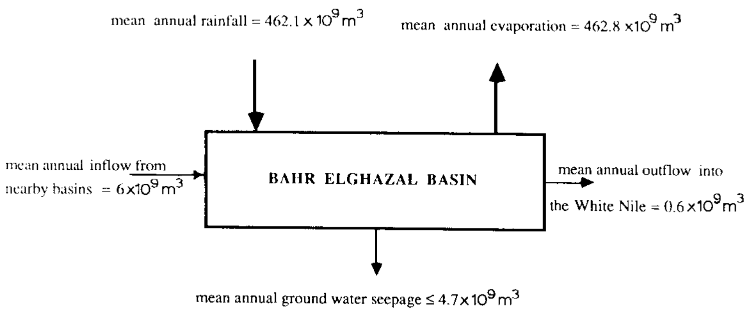

Fig. 3. Water balance of Bahr Elghazal basin (Chan and Eagleson, 1980)

Chan and Eagleson (1980) conducted a refined water balance for the basin. Their objective was to estimate the potential water recovery through drainage of the Bahr Elghazal swamps. The results are shown in Fig. (3). From these results it is clear that the water balance of the basin is dominated by two porcesses, rainfall and evaporation. The mean total annual evaporation from the basin is more than five times the mean annual flow of the Nile.

THE MATHEMATICAL MODEL THE FEEDBACK MECHANISM

A mathematical model was developed to represent the water balance of the atmospheric system above the Bahr Elghazal basin, the atmospheric system above Central Sudan and the water balance of the Bahr Elghazal basin. These systems are shown in Fig. (4). The water balance of these systems is based on the principle of conservation of mass.

The following notations were used in deriving the model equations: $I(t)$ = lateral transfer of water to the atmospheric system above the Bahr Elghazal basin; $O(t)$ = lateral transfer of water from the atmospheric system above the Bahr Elghazal basin; $R(t)$ = annual rainfall in Bahr Elghazal basin; $E(t)$ = annual evaporation from Bahr Elghazal basin; $DH(t)$ = difference between the water in the atmospheric system above the Bahr Elghazal basin at the end and the beginning of the year; and $DW(t)$ = difference between the surface storage in Bahr Elghazal basin at the end of the hydrological year and at the beginning of the year. The hydrological year begins of the 1st of November. The notation with primes refers to corresponding quantities for Central Sudan.

The water balance of the atmospheric system above the Bahr Elghazal basin is represented by:

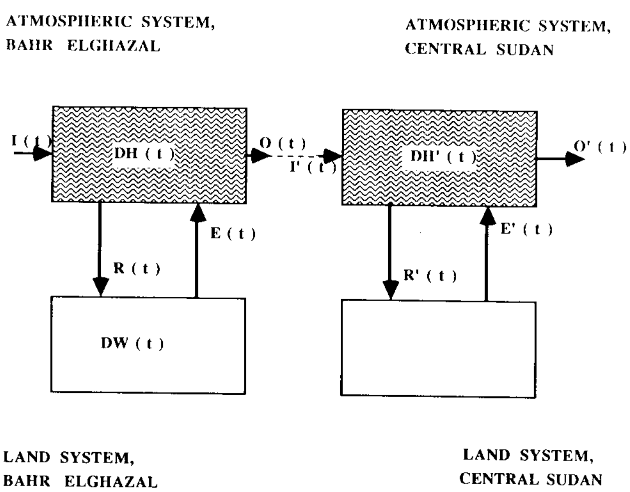

Fig. 4. Atmospheric and land systems, Bahr Elghazal and Central Sudan.

$$DH(t) = (t) + E(t) - R(t) \tag{1}$$

which can be rearranged as:

$$O(t) = I(t) + E(t) - DH(t) - R(t) \tag{2}$$

The water balance of the atmospheric system above Central Sudan is represented by:

$$DH'(t) = I'(t) + E'(t) - O'(t) - R'(t) \tag{3}$$

which can be rearranged as:

$$R'(t) = I'(t) + E'(t) - O'(t) - DH'(t) \tag{4}$$

Since the lateral transfer to the atmospheric system of Central Sudan comes from the south, $I'(t)$ is definitely related to $O(t)$:

$$I'(t) = F[O(t)] \tag{5}$$

Substituting for $I'(t)$ from eqn. (5) into (4) yields:

$$R'(t) = F[O'(t)] + E'(t) - O'(t) - DH'(t) \tag{6}$$

Substituting for $O(t)$ from eqn. (2) into eqn. (6) yields:

$$R'(t) = F[I(t) + E(t) - DH(t) - R(t)] + E'(t) - O'(t) - DH'(t) \tag{7}$$

The water balance of the Bahr Elghazal basin as shown in the previous section, is represented by:

$$DW(t) = R(t) - E(t) \tag{8}$$

Since the rainfall and evaporation are not equal for all the years, the evaporation in any year is a summation of proportions of rainfall in that year and previous years. It is suggested that this can be represented by:

$$E(t) = P_0(t)R(t) + P_1(t)R(t-1) + P_2(t-2) + \cdots + \tag{9}$$

where $P_i(t)$ is the fraction $R(t-i)$ evaporated in the year t, a function of time. For simplicity consider the case for which $P_i(t) = 0$ for all $i > 1$. Substituting for $E(t)$ from eqn. (9) into eqn. (7) gives:

$$R'(t) = F[I(t) + P_0(t)R(t) + P_1(t)R(t-1) - DH(t) - R(t)] + E'(t) - O'(t) - DH'(t) \tag{10}$$

Nicholson (1980) studied the annual rainfall series in the region of western and central Africa. She concluded that the annual rainfall series in the Sahel region, which included Central Sudan, is very similar to the annual rainfall series in the Soudan region, which included northern parts of Bahr Elghazal basin. The correlation coefficent between normalised annual rainfall departures from the two regions was calculated as 0.83. She also reported significant correlation between the Soudan–Guinean region, which includes southern parts of Bahr Elghazal basin and the Sahel region, which includes Central Sudan. The correlation coefficient was calculated as 0.21.

Based on the results of this study, it is assumed that annual rainfall in Bahr Elgahzl is related to annual rainfall in Central Sudan by:

$$R(t) = f[R'(t)]$$

or:

$$R(t-1) = f[R'(t-1)] \tag{11}$$

Substituting for $R(t-1)$ from eqn. (11) into eqn. (10) and simplifying yields:

$$R'(t) = F\{I(t) + [P_0(t) - 1]\,R(t) + P_1(t) f[R'(t-1)] - DH(t)\} + E'(t) - O'(t) - DH(t) \tag{12}$$

Equation (12) is the general equation which expresses mathematically the feedback mechanism in annual rainfall in Central Sudan. It can be simplified by the following assumptions:

(a) The output $O(t)$ is assumed equal to the input $I'(t)$ for two reasons. The regions east, south and west of the Bahr Elghazal basin are humid. Central Sudan is the only neighbouring dry region. Hence there is a net output of water vapour from these regions to Central Sudan. The system of winds which affects the entire region most of the year is southernly and brings all the water in the region from the oceans. Central Sudan is located to the north of Bahr Elghazal.

(b) Eltahir (1987) concluded that the linear relationship is suitable for the relation between annual rainfall in different stations in the region. Hence eqn. (11) is most likely a linear equation and of the form:

$$R(t) = a + b\,R'(t) + e(t) \tag{13}$$

where a and b are constants, and $e(t)$ a random component.

(c) $P_i(t)$ is assumed to be constant and independent of time. This assumption is discussed in the Appendix. Based on this assumption eqn. (9) can be rewritten as:

$$E(t) = P_0 R(t) + P_1 R(t-1) + P_2 R(t-2) + \cdots + \tag{14}$$

where P_i is the fraction of $R(t-i)$ evaporated in year t.

Based on these there assumptions, eqn. (12) can be rewritten as:

$$R'(t) = I(t) + P_1 a + P_1 b R'(t) - 1) + e''(t) - DH(t) + E'(t) - O'(t) - DH'(t) \tag{15}$$

where $e''(t)$ is a random variable, as a result of multiplying $e(t)$ by a.

This is the proposed equation for modelling the feedback mechanism in the region of Central Sudan. The variables $I(t)$ and $R(t)$ are nearly random, with the possibility of slight serial dependence, as a result of similar feedback mechanisms over similar swampy areas in equatorial Africa. These two random variables account for most of the variability in $R'(t)$. $R'(t-1)$ is the variable which expresses an explicit feedback mechanism. The term $(P_1 a)$ is nearly a constant. Hence $R'(t)$ has a random component and a weak dependence structure, similar to properties observed in annual rainfall series in Central Sudan.

CONCLUSIONS

It is likely that the high levels of evaporation from the Bahr Elghazal basin has a significant effect on the climates of the neighbouring dry regions. Any project which plans to introduce changes in the hydrology of the basin by reducing the evaporation losses should investigate into the possible reduction in the rainfall amounts in Central Sudan.

The annual rainfall process in Central Sudan has a weak dependence structure which makes it difficult to trace the caustive physical mechanisms. The above derivation suggests a possible mechanism, although it does not prove it. Further investigations may confirm the validity of the mechanism, but that needs more data, namely more accurate measurements of the regional rainfall amounts. Until such kinds of data are available the mechanism remains a reasonable hypothesis.

ACKNOWLEDGEMENTS

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APPENDIX—DISCUSSION OF ASSUMPTION (C)

Since rainfall and evaporation in Bahr Elghazal basin are not equal for all the years, the evaporation in any year is a summation of proportions of rainfall in the same year and previous years. It can be represented by the following equation:

$$E(t) = \sum_{i=0}^{\infty} P_i(t)\, R(t-i) \tag{A1}$$

where $P_i(t)$ is the fraction of $R(t-i)$ evaporated in year t. $P_i(t)$ is a function of the geometry of the storage system and drainage conditions. Most of the evaporation from Bahr Elghazal basin comes from swampy areas, hence the storage system is dominated by these swamps.

Sutcliffe and Parks (1987) studied the hydrology of the Sudd swamps, which is the neighbouring region to Bahr Elghazal basin. They assumed that the relation between the volume of water in the swamp, W, and the surface area of the swamp, A, can be represented by:

$$A(t) = KW(t) \tag{A2}$$

where K is a constant. A similar relationship was assumed by Hutton and Dincer (1979), while studying Okavango swamp in Botswana.

The same relation was assumed for the swamps in Bahr Elghazal region. A was defined as an equivalent area which would produce a total evaporation for that year equal to the one record. W is the corresponding volume of water available in the swamp and it is approximately equal to the average volume of water in the swamps for that year. Evaporation, E, from the swamps is represented by:

$$E(t) = PEA(t) \tag{A3}$$

where PE is the annual potential evaporation. Substituting for $A(t)$ from eqn. (A3) into eqn. (A2) yields:

$$E(t) = PEKW(t) \tag{A4}$$

Considering any two years and denoting the variables in these by suffixes 1 and 2, and rewriting eqn. (A4) in a finite difference form we get:

$$E_2 - E_1 = PEK(W_2 - W_1) \tag{A5}$$

The finite defference approximation for $W_2 - W_1$ is represented by:

$$W_2 - W_1 = \tfrac{1}{2}[(R_1 + R_2) - (E_1 + E_2)] \tag{A6}$$

Substituting for $W_2 - W_1$ in eqn. (A5) and rearranging we get:

$$E_2 = \frac{C}{(1 + C)}(R_1 + R_2) + \frac{(1 - C)}{(1 + C)}E_1 \tag{A7}$$

where C is given by:

$$C = \frac{PEK}{2} \tag{A8}$$

Similary for the year 3, 4 and 5 we can derive:

$$E_3 = \frac{C}{(1 + C)}(R_2 + R_3) + \frac{(1 - C)}{(1 + C)}E_2 \tag{A9}$$

$$E_4 = \frac{C}{(1 + C)}(R_3 + R_4) + \frac{(1 - C)}{(1 + C)}E_3 \tag{A10}$$

and:

$$E_5 = \frac{C}{(1 + C)}(R_5 + R_4) + \frac{(1 - C)}{(1 + C)}E_4 \tag{A11}$$

Combining eqns. (A7), (A9), (A10) and (11) we get:

$$E_5 = \frac{C}{(1 + C)}R_5 + \frac{2C}{(1 + C)^2}R_4 + \frac{2C(1 - C)}{(1 + C)^3}R_3 + \frac{2C(1 - C)^2}{(1 + C)^4}R_2 + \frac{C(1 - C)^3}{(1 - C)^4}R_1 + \frac{(1 - C)^4}{(1 + C)^4}E_1 \tag{A12}$$

Equation (A12) can be written in a general form similar to eqn. (A1) as:

$$E(t) = \sum_{i=0}^{\infty} P_i R\,(t - i) \tag{A13}$$

with the only difference that the fraction P_i is a constant, not a function of time.

To obtain some idea of the values of these fractions in the Bahr Elghazal basin, the value of K was assumed to be one (meter) ^{1}, obtained by calibrating the area-volume model in the neighbouring Sudd region. The potential evaporation was assumed to be 1.6 m, which is an average for the basin. According to eqn. (A8) the constant C was calculated as 0.8. The fractions of P_i were calculated according to eqn. (A12) and are:

P_0	P_1	P_2	P_3
0.44	0.49	0.05	0.006

From the values shown it is clear that the memory of the system is very short and the evaporation in any year is a function of the rainfall in that year and the previous year, mainly.

Equation (A1) is the exact equation representing the relation between evaporation and rainfall in Bahr Elgahzal basin. Under certain reasonable assumptions, the above derivation proves that eqn. (A13) is also a good approximation. Using either equations in developing the mathematical model will only have slight effect on the final result.

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