Estimation of the Fractional Coverage of Rainfall in Climate Models

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ABSTRACT

The fraction of the grid cell area covered by rainfall, μ, is a very important parameter in the descriptions of land surface hydrology in climate models. A simple procedure is presented for estimating this fraction, based on extensive observations of storm areas and rainfall volumes. It is often observed that storm area and rainfall volume are linearly related. This relation is utilized in rainfall measurement to compute rainfall volume from radar observations of the storm area. The authors suggest that the same relation be used to compute the storm area from the volume of rainfall simulated by a climate model. A formula is developed for computing μ, which describes the dependence of the fractional coverage of rainfall on the season of the year, the geographical region, rainfall volume, spatial resolution of the model, and the temporal resolution of the model.

The new formula is applied in computing μ over the Amazon region. Significant temporal variability in the fractional coverage of rainfall is demonstrated. The implications of this variability for the modeling of land surface hydrology in climate models are discussed.

1. Introduction

Spatial variability of rainfall is a very important factor in the description of hydrologic processes such as interception and runoff over large areas. Hence, recent parameterizations of hydrologic processes in climate models, for example, Warrilow et al. (1986), Shuttleworth (1988), Entekhabi and Eagleson (1989), Famiglietti and Wood (1990), and Eltahir and Bras (1991) include explicit representations of the rainfall spatial variability at the subgrid scale. In all these schemes rainfall is described as a random variable which varies in space covering a prescribed fraction of the grid cell area, μ. It is not resolved how to specify the value of this fraction for the different regions, in the different seasons and whether μ should vary during the life cycle of a single storm.

Several recent studies have focused on the sensitivity of modeling of large-scale surface hydrology to the value of μ. Johnson et al. (1991) used a climate model to study the sensitivity of land surface hydrology to the value of μ. They performed a control run for three years with μ equal to 0.6; then μ is reduced from 0.6 to 0.15 and the simulation is repeated for another three years. As a result, the runoff coefficient for South America increased from 0.13 to 0.44 and the runoff coefficient for Africa increased from 0.10 to 0.45. Similar large sensitivity of land surface hydrology was demonstrated by Pitman et al. (1990) and Thomas and Henderson-Sellers (1991). It seems that the choice of μ affects significantly the hydrology in climate models and hence, it should be done more carefully.

In this paper we suggest a simple and consistent procedure for computing μ, based on extensive observations of convective storms. These observations are reviewed in the next section. The theoretical basis that may explain these empirical observations is reviewed. Then we introduce a new procedure for computing μ. The procedure is applied in estimation of the fractional coverage of rainfall over the Amazon region using areally averaged rainfall simulated by a climate model.

2. Empirical observations

In dealing with the problem of estimation of the fractional coverage of rainfall we refer to the large number of observations collected and analyzed by another section of the meteorological community, namely the rainfall measurement and estimation group. These observations, Doneaud et al. (1981, 1984), Lopez et al. (1983, 1989), Chiu (1988), Kedem et al. (1990), and Johnson and Smith (1990), point to the remarkably high correlation between rainfall volume and storm area. The rainfall volume is defined as the volume obtained by integrating rainfall rate in space from a single snapshot and then multiplying by the temporal resolution of the measurement. Storm area is defined as the corresponding area which receives rainfall greater than zero.

Figure 1 is a compilation of some of the observations of rainfall volumes and storm areas. It is consistently observed that the relation of rainfall volume to storm

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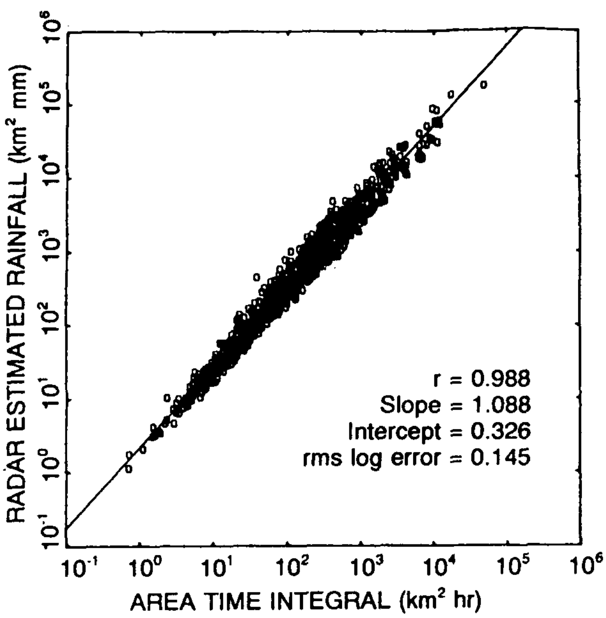
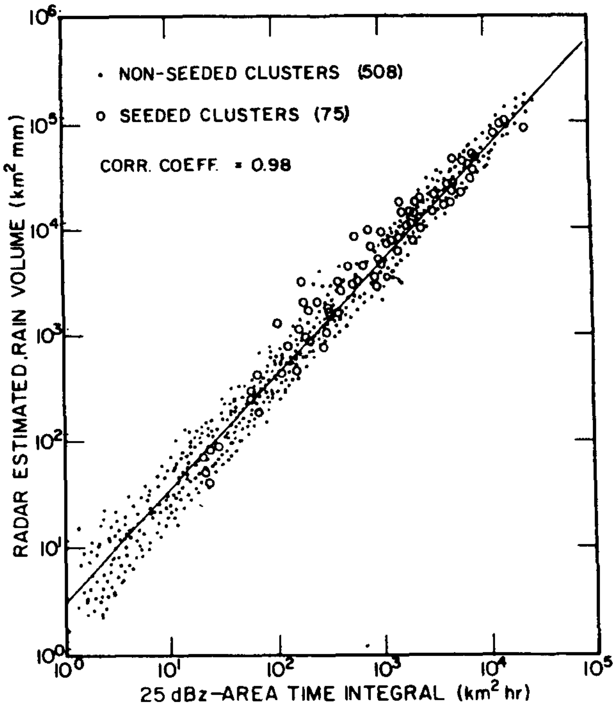
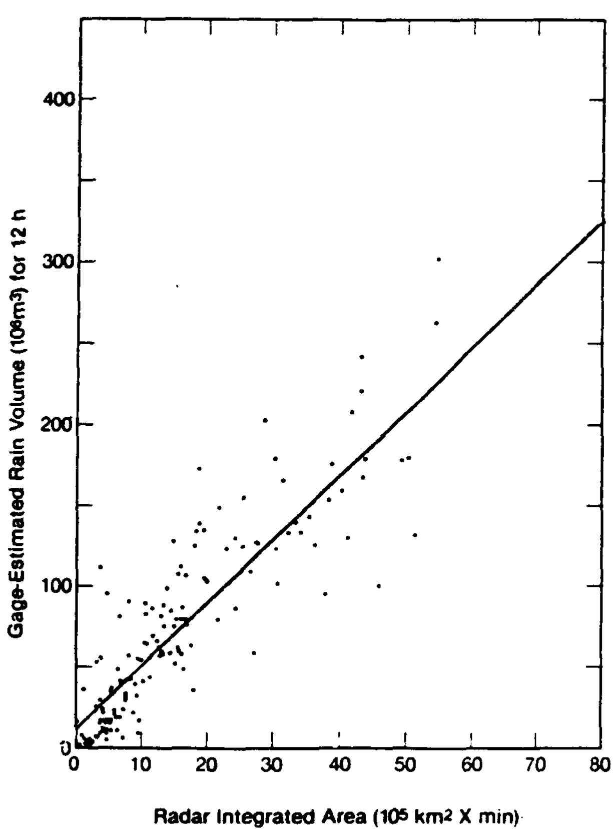
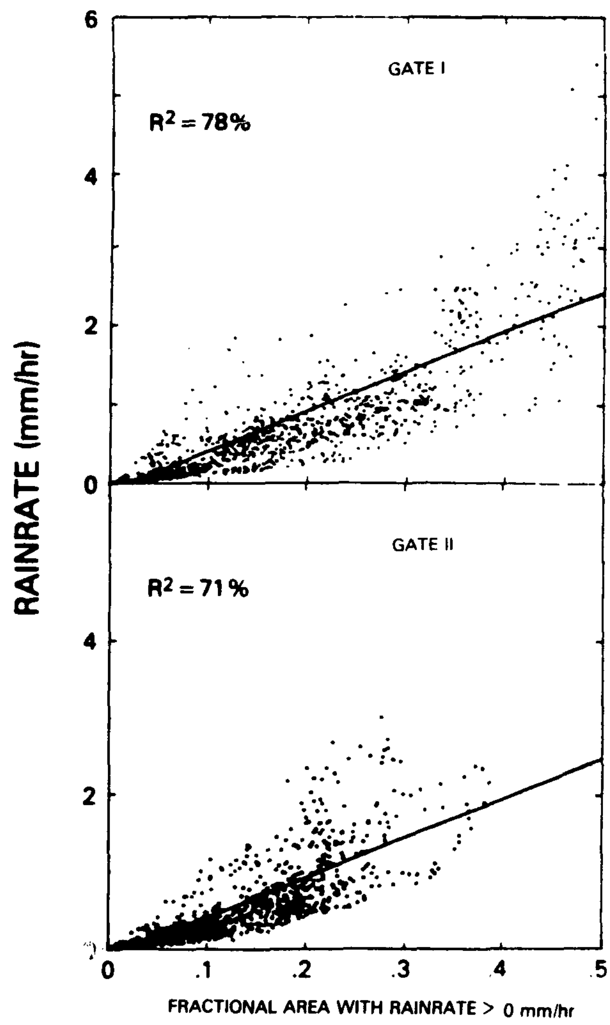

RADAR ESTIMATED RAINFALL (km² mm)
AREA TIME INTEGRAL (km² hr)
r = 0.988
Slope = 1.088
Intercept = 0.326
rms log error = 0.145


NON-SEEDED CLUSTERS (508)
SEEDED CLUSTERS (75)
CORR. COEFF. = 0.98
RADAR ESTIMATED RAIN VOLUME (km² mm)
25 dBz–AREA TIME INTEGRAL (km² hr)


Gage-Estimated Rain Volume (10⁶m³) for 12 h
Radar Integrated Area (10⁵ km² X min)


GATE I
R² = 78%
GATE II
R² = 71%
RAINRATE (mm/hr)
FRACTIONAL AREA WITH RAINRATE > 0 mm/hr

area is close to being linear. These observations describe convective storms in the tropics, the subtropics, and midlatitudes. Because of all these observations, the volume–area model is one of the most accurate techniques for estimation of rainfall volumes from space.

3. Theoretical basis

When rain falls over a fraction μ of a large area, the rainfall rate over the total area can be described statistically by the following mixed distribution:

$$g_R = (1 - \mu)\delta(R - 0) + \mu f_R, \qquad (1)$$

where δ is the Dirac delta function and f_R is the conditional probability density function (PDF) of the rainfall rate, R, given that R is greater than zero. Since no assumptions are made about the conditional PDF, f_R, the description in Eq. (1) is general and always valid.

For the rainfall rate process that has a *unique* conditional PDF, f_R, it is shown by Kedem et al. (1990) that the total volume of rainfall is linearly related to the area of the storm which receives rainfall rate above a certain threshold. When the value of this threshold is zero, the theory predicts the observed linear relation between the rainfall volume and storm area. (Similar linear relation will be developed in the next section.) A unique PDF means that f_R does not vary in time, that is, whenever it rains, the distribution of rainfall rate within the raining area is a realization from the same statistical distribution.

The assumption of a unique conditional PDF, f_R, is an idealization of the rainfall rate process. In the real world f_R may vary between the different storms and even within the life cycle of a single storm. Kedem et al. (1990) suggest that the relation between the total volume of rainfall and the area of the storm which receives rainfall rate above a certain threshold is robust when the threshold chosen is greater than zero. However the observations in Fig. 1 indicate that although f_R may vary in time, the range of this variability is small and, hence, the assumption of a *unique* f_R is a good working assumption.

4. Estimation of μ

The observed relation between storm area and rainfall volume is often used in estimating rainfall volume from the radar measurement of storm area. We suggest that the same relation be used, but this time to infer the storm area from the rainfall volume simulated by a climate model.

We make the assumption that the rainfall rate process is described by a unique conditional PDF, f_R. The mean of this conditional distribution is denoted by ρ and has a seasonal and geographical variability. The expected value of the rainfall rate process over the grid cell area of a climate model is given by

$$E(R) = \int_{R=0}^{\infty} R g_R dR$$

$$= (1 - \mu)0 + \mu \int_{R=0^+}^{\infty} R f_R dR = \mu\rho, \qquad (2)$$

implying that

$$\mu = \frac{E(R)}{\rho}. \qquad (3)$$

According to Eq. (3), slope in the regression between $E(R)$ and μ should be equal to ρ. The slope of the regression line in Fig. 1d is about 4.4 mm h^{-1}. This value is very close to the observed conditional mean rainfall rate in the Global Atmospheric Research Program Atlantic Tropical Experiment (GATE) experiment. Hence, Eq. (3) is consistent with the observations of Fig. 1.

Recalling that a climate model computes $E(R)$ from the following relation,

$$E(R) = \frac{V}{(\Delta X)^2 \Delta T}, \qquad (4)$$

where V is the volume of rainfall simulated by the climate model within a grid cell area and ΔX and ΔT are the spatial and temporal resolutions of the model. Substituting for $E(R)$ from Eq. (4) into Eq. (3) results in

$$\mu = \frac{V}{(\Delta X)^2 \Delta T \rho}, \quad \mu \leqslant 1.0. \qquad (5)$$

By definition, μ is restricted to the range of values between zero and one.

Equation (5) is the formula for computing the fractional coverage of rainfall. It incorporates most of the important factors controlling μ. The regional climate is represented by the conditional mean of the rainfall rate process. The spatial and temporal resolutions of the climate model appear explicitly in Eq. (5). The volume of rainfall simulated by the model during a time period ΔT varies between the different storms and within the life cycle of the same storm. According to the linear relation in Eq. (5) μ should vary similarly. This variability is consistent with the observations of convective storms; the raining area initially increases during the early development of a convective storm

FIG. 1. Observations of rainfall volume and storm area. (a) Cooperative Convective Precipitation Experiment (from Johnson and Smith 1990), (b) the Bowman dataset (from Doneaud et al. 1984), (c) Florida Area Cumulus Experiment (from Lopez et al. 1989), (d) GATE dataset (horizontal axis is the storm area divided by the observed area; vertical axis is rainfall volume divided by the observed area. The observed area has a diameter of 400 km) (Kedem et al. 1990).

and then slowly decreases while the storm dissipates. The seasonal variability of ρ is reflected in the estimate of μ through Eq. (5). Figure 2 illustrates with a simple example the dependence of μ on the conditional mean of the rainfall rate process, the rainfall volume, the model spatial resolution, and the model temporal resolution.

The observations and the theory described in the previous sections are for convective storms. Hence, the procedure introduced in this paper is more accurate for modeling convective storms. Under those conditions, it should be very rare that μ approaches a value of 1. During the GATE experiment, which was conducted over the tropical ocean, the maximum observed value for μ is about 0.5 (see Fig. 1d). The observed area has a diameter of 400 km. According to Eq. (5) the value of μ is not allowed to exceed 1.

For nonconvective storms associated with frontal systems, the fractional coverage of rainfall approaches a value of 1 more often than for convective storms (depending on the relative size of the grid cell compared to the typical area of the storm). The use of Eq. (5) for estimation of the fractional coverage of rainfall in these storms may result in estimates of μ that exceed 1. That corresponds to the possibility of occurrence of a frontal storm covering the total grid cell area and with an average rainfall rate which exceeds the conditional mean rainfall rate. Under those conditions μ should be reset to a value of 1.

The formula for computing μ is developed primarily for convective storms. It is less accurate when used for estimation of μ in nonconvective storms. We believe it provides a better approximation than the assumption that $\mu = 1$ for these kind of storms since that assumption is often made independent of the model spatial resolution and the rainfall volume. This is particularly true when the model resolution is large compared to the typical scale of a frontal storm (a few hundred kilometers).

5. Estimation of ρ

Only one parameter is required in the above procedure, namely, the conditional mean rainfall rate, ρ. We assume that the conditional distribution of the rainfall rate process is ergodic. Hence, ρ can be esti-

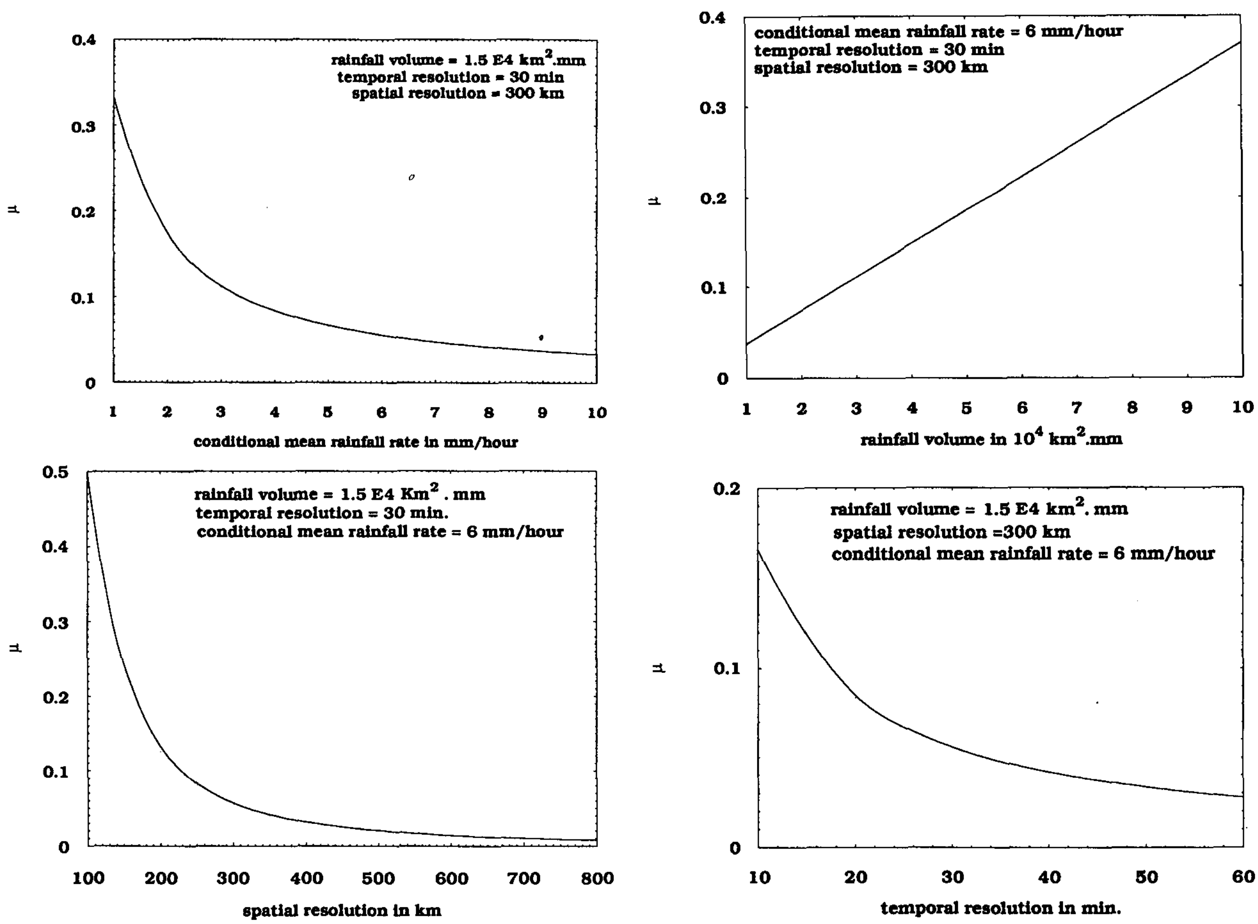


FIG. 2. Illustration of the dependence of μ on (a) conditional mean rainfall rate, (b) rainfall volume, (c) spatial resolution, and (d) temporal resolution.

mated by the climatological mean rainfall rate at a single location. This estimate is consistent with the assumption of a unique conditional PDF, f_R. According to this assumption every snapshot in every storm is a realization of the same statistical distribution. Invoking the ergodicity assumption, the mean of the conditional distribution can be estimated by the mean of the rainfall rate process at a point when computed from a sufficiently large number of these realizations.

An estimate of ρ is taken from the rainfall records at a single location and for each month, M. It is estimated by

$$\rho(M) = \sum_{I=1}^{N} r(I, M) / \sum_{I=1}^{N} t(I, M), \qquad (6)$$

where N is the total number of years with recorded rainfall amounts, r the monthly total rainfall amount, and t the monthly total duration of storms. Table 1 shows estimates of ρ from different regions and for the different months of the year. The estimated values for ρ are larger in the tropics compared to midlatitudes, and at each location those estimates are larger in summer as compared to winter. These observations are consistent with the differences in the rainfall-producing mechanisms. The information in Table 1 is sufficient for modeling μ at those locations.

6. Application in the Amazon region

The procedure introduced in the previous sections is applied in computing the fractional coverage of rainfall over the Amazon region. A time series of rainfall, averaged over a grid cell area, is simulated by the climate model of the National Center for Atmospheric Research (NCAR) (CCM1). The location of the grid cell corresponds to the Amazon region. The spatial resolution of the model is (approximately) 4.4° lat by 7.5° long. The temporal resolution of the series is one-half hour and the period covered is the first 300 days of a typical year. The same rainfall series is used in the study by O'Neill and Dickinson (1991).

Figure 3 shows the time series of μ which is obtained by applying Eq. (3) to the rainfall series from the Amazon region. The conditional mean rainfall rate is estimated by the climatological mean rainfall rate at Manaus (see Table 1). The variability in μ is quite significant; μ is as variable as the areal average of rainfall. Parameterizations of surface hydrology in climate models often assume that μ is a constant, for example, currently the U.K. Meteorological Office (UKMO) climate model assumes that μ is 0.3 for convective rainfall. The significant variability in Fig. 3 raises many questions about the accuracy of the current descriptions of land surface hydrology in climate models.

7. Concluding remarks

The procedure introduced in this paper captures the seasonal and geographical variability in the fractional coverage of rainfall, μ. It even describes the variability of μ from storm to storm and within the life cycle of a single storm. Another important advantage of our formula for computing μ is the ability to describe explicitly the dependence of the fractional coverage of rainfall on the spatial and temporal resolutions of the climate model.

The estimation procedure presented in this paper is easy to apply. Instead of specifying a constant value for μ to characterize rainfall in each region of the world, we suggest that the climatological mean rainfall rate in every region be computed and μ be allowed to vary in space and time. It is much easier to obtain information about the climatological mean rainfall rate from the records at a single raingage than to obtain information about the fractional coverage of rainfall.

From the results of applying our procedure in computing μ over the Amazon Basin, we conclude that inclusion of the variability of μ in land surface hydrology parameterizations is crucial to the accuracy of those descriptions. It is unreasonable to neglect the effects of spatial variability on land surface hydrology over large areas, but it is equally unreasonable to assume that all the convective storms, in every region of the world, and in every season, cover the same area.

TABLE 1. Climatological mean rainfall rate (in mm h^{-1}) from different regions of the world and for different months of the year.

Month	Wau (7°N, 28°E)	Manaus (3°S, 60°W)	Florence (44°N, 11°E)	Boston (42°N, 71°W)	Tucson (32°N, 111°W)
Jan	7.3	5.5	1.1	1.2	1.1
Feb	6.1	6.1	1.3	1.3	1.0
Mar	6.0	6.5	1.4	1.3	1.0
Apr	9.2	6.5	1.3	1.3	1.2
May	9.7	6.1	1.7	1.4	1.0
Jun	11.1	3.9	2.1	1.7	1.6
Jul	10.5	4.3	3.4	2.0	2.3
Aug	10.9	3.8	3.3	2.2	2.4
Sep	10.0	3.8	2.8	1.8	2.5
Oct	9.6	5.6	2.1	1.6	1.7
Nov	6.0	5 3	2.0	1.6	1.2
Dec	6.6	6.6	1.3	1.3	1.2

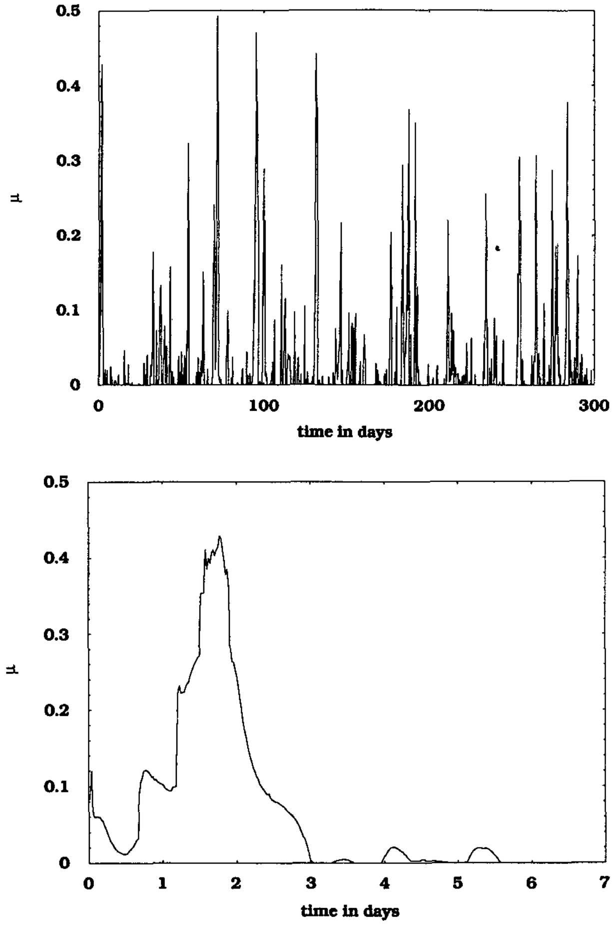


FIG. 3. Fractional coverage of rainfall in the Amazon region (a) the first 300 days of a typical year, (b) the first week of the year.

The procedure presented in this paper is a solution for that problem, consistent with the observations (see Fig. 1), which demonstrate that μ is indeed a variable and not a constant parameter.

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