On the origin of near uniformity of the spatial distribution of velocity in natural rivers

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Abstract

Downstream hydraulic geometry equations provide insight into the equilibrium tendencies of natural rivers. Numerous field studies have demonstrated that channel cross-section averaged velocity is nearly uniform in a downstream direction. Here, we present a simple theoretical framework that attempt to explain the near uniformity of average velocity in the downstream direction for basins that are near equilibrium with respect to sediment fluxes. Assuming that at equilibrium, tractive force, or average boundary shear, is uniform throughout a river network we predict that channel hydraulic radius scales with discharge to the $-\theta$ power, where θ is the scaling exponent in the corresponding relationship between channel slope and discharge. Assuming flow is locally uniform, we apply the Manning equation to predict that average velocity scales with discharge to the $-\theta/6$ power. This predicted exponent would typically fall in the range between 0.05 and 0.1 which implies near uniformity in the spatial distribution of velocity in natural rivers. We compare predictions from this theory to field observations.

(Key Words: spatial distribution velocity, hydraulic geometry, basin geomorphology)

1. Introduction

Local channel cross-section properties such as flow width, depth, and velocity, as well as channel bed-slope have frequently been parameterized as power functions of discharge. Such an approach has been used to describe these properties locally for varying discharges, and downstream for a discharge of constant frequency or geomorphic significance. The former approach has come to be known as at-a-station hydraulic geometry while the latter parameterization was termed downstream hydraulic geometry by *Leopold and Maddock* [1953]. The downstream hydraulic geometry equations most frequently discussed are,

$$w = c_0 Q^b \tag{1}$$

$$h = c_1 Q^f \tag{2}$$

$$u = c_2 Q^m \tag{3}$$

$$S = c_3 Q^\theta \tag{4}$$

where w is the channel top width, h is the hydraulic depth, u is the cross-section averaged velocity, S is the local channel bed-slope, and Q is discharge. It follows from continuity that the coefficients, c_0, c_1, and c_2 must multiply to unity, while the exponents, b, f, and m must sum to unity. Although a degree of non-linearity exists between upstream drainage area, A, and discharge [*Knighton*,1998], we assume A serves as a surrogate for a channel-forming discharge. Equations (1) through (4) can then be reformulated with A replacing Q. Figure 1 is a conceptual diagram of the downstream hydraulic geometry equations, and the meanings of their exponents. Within the context of these hydraulic geometry equations, throughout the remainder of the paper we will use the terms channel forming discharge and drainage area interchangeably.

Numerous field studies have quantified downstream hydraulic geometry parameters for varying hydroclimatic, geologic, and tectonic contexts [e.g., *Lacey*, 1929; *Leopold and Maddock*, 1953; *Nixon*, 1959; *Brush*, 1961; *Simons and Albertson*, 1963; *Langbein*, 1964; *Kellerhals*, 1967; *Emmett*, 1975; *Rundquist*, 1975; *Charlton et al.*, 1978; *Mahmood, et al.*, 1979; *Bray*, 1982; *Andrews*, 1984; *Hey and Thorne*, 1986; *Montgomery and Gran*, 2001; *Merritt and Wohl*, 2003].

Although variability in the magnitude of the parameter b in Equation (1) exists, b tends toward a value of 0.5, while f is usually observed to be around 0.4. These field investigations for networks representing a range of conditions has shown that m typically varies between 0.08 and 0.20. In a notable study, *Pilgrim* [1977] summarized values of m for seven field studies throughout the United States. The absolute magnitude of m indicates a relatively mild dependence of cross-section averaged velocity to discharge in a downstream direction, while when compared to b and f, the magnitude of m indicates that changes in channel width and depth downstream are relatively more dramatic than changes in cross-section averaged velocity.

Previous work attempting to predict the magnitudes of the downstream hydraulic geometry parameters theoretically and explain their remarkable consistency in varying hydroclimatic, tectonic, and sedimentary environments have typically adopted one or more underlying principles governing channel network self-organization. For instance, *Singh et al.* [2003] derive four classes of downstream hydraulic geometry equations based on principles of maximum entropy production and minimum energy dissipation in the channel network. *Huang, et al*. [2002] assume the principle of least action to derive stable channel dimensions for alluvial channels. *Julien and Wargadalam* [1995] conducted a three-dimensional stability analysis for noncohesive sediments, solving four governing equations to define the hydraulic geometry relations of alluvial rivers. *Li et al*. [1976] used equations describing channel morphologic response in an attempt to derive both at-a-station and downstream hydraulic geometry equations. *Parker* [1979] used an approach of downstream momentum transfer to resolve the paradox of a stable channel with an active bed, while *Chang* [1980] applied the principle of minimum stream power in other theoretical efforts to the derive the hydraulic geometry relations. In their landmark work on downstream hydraulic geometry, *Leopold and Maddock* [1953] used a minimum variance approach to derive the magnitudes of the downstream hydraulic geometry exponents.

Here, we present a relatively simple theoretical framework to explain the relatively mild dependence of cross-section averaged velocity on discharge in a downstream direction for homogeneous basins, making only an assumption of basin equilibrium. For such basins, we argue that the near uniformity in the spatial distribution of cross-section averaged velocity arises because of the offsetting effects of increasing discharge downstream and channel cross-sectional geometry adjustment to convey the associated increase in sediment load.

2. Theory

2.1 Theoretical framework

In nature, m in Equation (3) has been observed to fall in a relatively narrow range between approximately 0.08 and 0.20 which suggests that cross-section averaged velocity depends only mildly on discharge in a downstream direction. Field and theoretical studies demonstrate that this observation holds across varying hydroclimatic and geologic settings and for a range of channel bed and bank roughness (Table 1). Here we outline a conceptual framework to explain the mild dependence of channel velocity to discharge in a downstream direction.

We consider a conceptual watershed exhibiting homogeneous sedimentary material that exists in an equilibrium which we define to be a state in which the sediment carried at any location along the channel network is equal to the material delivered to the basin through tectonic uplift. At any point along the channel network the change in elevation can be described as [e.g., Kirkby, 1971],

$$\frac{dz}{dt} = U - f_S , \tag{5}$$

where z is the elevation, U is the tectonic uplift rate, and f_s is the flux per unit watershed area of sediment out of the basin. F_s is comprised of the sediment delivered in the channel from upstream and the sediment removed through local erosion. For a network in equilibrium the left-hand side of Equation (5) is zero and the outgoing sediment flux is balanced by input due to tectonic uplift at any point in the network. Then for transport limited conditions, in terms of volumetric sediment fluxes, the outgoing sediment flux along the channel network is equal to the fluvial sediment transport capacity while the amount of material input to the channel is the time-invariant spatially uniform uplift rate integrated over the watershed area. In a numerical landscape evolution model *Willgoose* [1991] expressed the capacity to transport sediment in the channel as a function of channel slope and upstream drainage area,

$$UA = \beta S^n A^p , \tag{6}$$

where β is a proportionality constant, and n and p are exponents. If the uplift rate, U, is constant in time and uniform in space then the channel slope obeys power law scaling in area [*Willgoose*, 1991],

$$S \sim A^{\frac{1-p}{n}} \sim A^{\theta} \; . \tag{7}$$

Note that Equation (7) is a restatement of Equation (4), assuming that the contributing basin area is a surrogate fluvial discharge. We will refer to θ as the slope-area parameter. The behavior predicted by Equation (7) has been observed in channel networks in a variety of geologic and hydroclimatic setting [e.g., *Tarboton et al*., 1989]. Furthermore, *Rodriguez-Iturbe and Rinaldo* [1997] discuss the implications of the observed power-law slope-area scaling in natural river networks.

Since U is assumed constant in time and uniform in space, Equation (6) is valid everywhere in the channel network and by extension every point in the channel network is at equilibrium. Therefore, at any cross-section along the channel network there can be no net erosion/deposition of sediment.

A homogeneous watershed not in equilibrium will exhibit locations within where erosive forces systematically exceed resisting forces. Given sufficient time and stationary driving forces, cross-sectional width, depth, and, by association, the average velocity will adjust at each location along the channel network such that the constraint of no net erosion or deposition is satisfied. In an engineering context, *Chow* [1959] envisioned the design of stable channels to be a balance between the tractive force and the permissible tractive force. The latter is the maximum erosive force that a cross-section can sustain without erosion of the channel margin. For homogeneous watersheds approaching equilibrium, we assume that the channel network tends toward a state in which the tractive force, or boundary shear,

$$\tau = \gamma RS , \tag{8}$$

is constant everywhere in the network. Persistent departure from this condition at any location within the network should lead to transient landscape evolution towards equilibrium. In Equation (8) τ is the tractive force or average boundary shear stress, R is the hydraulic radius, and S is taken as the local channel bed-slope.

Although channel material is almost certainly non-uniform along natural rivers [e.g., *Rice*, 1994], work by *Pitlick and Cress* [2002] demonstrates only a mild downstream change in bankfull shear stress for a 100 km reach of the Colorado River which exhibits a mild downstream fining trend. They also observed bankfull dimensionless shear stress, τ_b^*, to be less dependent on along-stream distance than bankfull shear stress over the same reach. τ_b^* is the average shear at the bankfull discharge normalized by a characteristic grain size, d_s, and is computed as $(R_b S)/(1.65 d_s)$ for quartz sediment, where R_b is the hydraulic radius at the bankfull discharge. Figures 8 and 10 in the work of *Pitlick and Cress* [2002] seems to support the assertion that, channel cross-section adjustment occurs in such a way that shear stress tends to remain constant in a downstream direction.

If Equation (8) holds in approximation throughout a channel network, substituting Equation (7) into equation (8), we find that the hydraulic radius must exhibit power law scaling with drainage area,

$$R \sim A^{-\theta}. \tag{9}$$

We further assume that flow is locally uniform at each point along the channel network, and that the cross-section average velocity can be related to the hydraulic radius and bed-slope through the Manning equation,

$$u = \frac{1}{n_0} R^{2/3} S^{1/2}, \tag{10}$$

where n_0 is the Manning roughness coefficient. Assuming that the flow resistance is due only to grain roughness we treat n_0 as a parameter which does not vary in a downstream direction because the bed material is assumed uniform throughout the network. We can then consider the downstream trend in velocity by substituting Equations (7) and (9) into (10). This yields,

$$u \sim \left(A^{-\theta}\right)^{2/3} \left(A^{\theta}\right)^{1/2} \sim A^{-\theta/6}. \tag{11}$$

Equation (11) expresses the downstream change in a local hydraulic property as a function of basin-scale geomorphic attributes. The theoretical framework developed here suggests the increase in discharge in a downstream direction is nearly balanced by an adjustment in cross-sectional width, depth, and slope such that there is no net erosion or deposition at the channel-forming discharge. The net effect of these trends in discharge and channel cross-section adjustment downstream is a relative insensitivity of cross-section averaged velocity to discharge.

Note that to the extent that there is a weak trend in velocity downstream, if shear stress is constant throughout the network, then the product of shear stress and velocity, which defines specific stream power, should also be weakly dependent on location within the channel network. Hence our approach differs from the concept of a uniform distribution of stream power assumed in the derivation of the optimal channel network framework set forth by *Rodriguez-Iturbe et al.* [1992], and applied and tested by *Molnar and Ramirez* [1998a,b] for a small watershed in Mississippi.

2.2 Methods of comparison to observed data

Several implications of the theoretical framework described in the previous section allow us to compare this framework against observations to test the validity of the framework. Equation (9) suggests that hydraulic radius scales with the negative of the slope-area parameter in a downstream direction. By definition, the channel hydraulic radius is a function of local channel properties,

$$R = \frac{A_{cs}}{P}, \tag{11}$$

where A_{cs} is the channel cross-section area, and P is the wetted perimeter. For assumed cross-sectional geometries, the hydraulic radius can be expressed explicitly as a function of the top width, w, and the flow depth, h. The three most simple cross-sectional geometric configurations are rectangular, triangular, and trapezoidal geometries, which can be expressed as functions of the top width and flow depth as follows:

Rectangular:

$$R = \frac{wh}{w+2h}. \tag{12}$$

Triangular:

$$R = \frac{wh}{4\sqrt{1/4\,w^2+h^2}}. \tag{13}$$

Trapezoidal:

$$R = \frac{\left(w - z_{ss}h\right)h}{w - 2z_{ss}h + 2h\sqrt{1+z_{ss}}} \tag{14}$$

z_{ss} in Equation (14) is the inverse of the channel side-slope. Further, for wide rectangular channel cross-sections (i.e., $w \gg h$) $R \approx h$.

Substituting the hydraulic geometry equations for flow width and depth (Equations (1) and (2)) into the equations for the hydraulic radius of an assumed cross-sectional shape (Equations (12)-(13)) we can determine how hydraulic radius should scale with area according to field observations. For example, we immediately see that for wide, rectangular cross-sections,

$$R \approx h \approx c_1 A^f, \tag{15}$$

and,

$$\theta \approx -f. \tag{16}$$

The exponent of this scaling relationship represents an estimate θ obtained independently of the slope-area parameter itself, which is compared to the field measured values of θ from available field studies.

Equation (11) implies that our theoretical description relates the velocity equation exponent, m, to the slope-area parameter, θ, as,

$$m = -\frac{\theta}{6}. \tag{17}$$

The previously described estimate of θ obtained from the hydraulic geometry equations for flow width and depth, and assumed cross-sectional geometries are then used to estimate the velocity equation exponent, m. These estimates of m are also compared to published values in the following section.

3. Comparison with observations

Values of the exponents observed through field measurement, b_o and f_o, from seven field studies of natural rivers were used to compute values of θ, denoted θ_c, by assuming one of three different cross-sectional geometries and then substituting Equations (1) and (2) into the appropriate equation for the hydraulic geometry (Equations (12)-(14)). Comparing θ_c to values of θ obtained directly through field inspection, θ_o, we find that the θ_c are able to approximate θ_o relatively well, although there are notable counterexamples to this finding (Table 2). We also note that θ_c appears more sensitive to the magnitude of f_o than to the magnitude of b_o. Using these same field data, we computed values of m, denoted m_c, by substituting the values of θ_c shown in Table 2 into Equation (17). When compared to the values of m measured directly from field inspection, m_o, the values of m_c typically underestimate the measured values by a factor of approximately two (Table 3). In the following section, we describe a few probable reasons for observed discrepancies between θ_c and θ_o, as well as m and m_o.

4. Discussion and Conclusions

The theoretical framework we have outlined in this paper made several simplifying assumptions regarding the state of a basin being considered. The conceptual system for which we develop the theoretical framework is in an equilibrium in which the material delivered through tectonic uplift is balanced by the sediment eroded from hillslopes and transported out of the channel network. Such an equilibrium represents a balance between the climatic and tectonic forces driving landscape evolution and the resistance of the material that composes the basin. *Whipple* [2001] argues that estimated minimum time scales of landscape response to climatic and tectonic perturbations are longer than the observed timescales of climatic fluctuation in the Quaternary, making the equilibrium topographies assumed here unlikely in natural landscapes.

We also assume that sediment size and erodibility is uniform throughout space and constant in time, and that sufficient supply of sediment exists at all times to satisfy the sediment transport capacity at any point in the basin. The sediment transport capacity relation presented in Equation (6) does not account for threshold dependent detachment-limited behavior, however, which is likely to dominate natural landscapes [*Howard*, 1994]. Detachment limited behavior would presumably change the values of n and p in Equation (6), yielding a different value of the slope-area parameter.

To obtain Equation (7) by solving Equation (6) explicitly for the channel slope, we made the assumption that the tectonic uplift was uniform in space and constant in time. Observational evidence suggests that tectonic uplift rates can be episodic through time and spatially nonuniform [e.g., *Wegmann and Pazzaglia*, 2002]. Episodic and spatially-varying tectonism would tend to decrease the likelihood that a landscape would attain an equilibrium state.

Channel networks in mountain drainages often demonstrate significant downstream trends in grain size [e.g., *Rice*, 1994] and, by extension, grain resistance. Downstream trends in streambed substrate size have also been shown to be punctuated at tributaries junctions and at local sites of coarse colluvium input associated with landslides and debris flows [*Rice*, 1998]. Significant changes in substrate size distribution along a river may be associated with significant non-uniformity in values of critical shear stress. Departures from uniform spatial distributions of shear stress would imply that R scales with area to some power different from θ. Even with uniform bed material and grain roughness in a downstream direction, flow resistance is likely to change downstream due to changes in form roughness associated with meandering, secondary flows, or large woody debris [*Leopold et al.*, 1964]. Either of these two cases would have the effect of making n_0 in Equation (9) vary along a channel, meaning that the downstream trend in velocity would scale with area to some power different from $-\theta/6$. For changing roughness associated with punctuated downstream fining or large woody debris, the channel roughness along the channel network would be a discontinuous function of drainage area.

Cross-section averaged velocity scales with discharge to the $-\theta/6$ power only when flow is locally uniform, a condition which arises when the gravitational gradient and frictional resistance are in balance. For realistic values of θ, the assumption of uniform flow in the network predicts that m varies between approximately 0.05 and 0.10. Yet, observations suggest that m is closer to being in the range of 0.10 to 0.15, and therefore that a degree of local non-uniformity in open channel flow exists and can account, in part, for the discrepancy between observed and predicted values of m.

Downstream hydraulic geometry parameters in Equations (1)-(4) are typically determined through field measurement of the flow width and depth at a geomorphically significant event, such as the bankfull discharge. The magnitude of this discharge at each location along a river is frequently obtained directly through known stage-discharge relationships, computed through an equation similar in form to the Manning equation, or estimated flood frequency analysis. The cross-section averaged velocity corresponding to this discharge is then computed through the equation of continuity. Hydraulic geometry parameters are then determined by regressing the flow width, depth and velocity against either the discharge at which they are applicable or the upstream contributing area at their location of measurement. Field determination of a geomorphically significant discharge in natural channels is made difficult by a lack of, or inconsistency of field evidence to quantify the flow width and depth, and a corresponding roughness for this discharge, if it is required. Determination of discharge magnitudes from flood-frequency analysis, on the other hand, relies on flow records of finite duration that are sometimes missing observations, assumptions of the return period of an event of geomorphic significance, and simplifying statistical assumptions about climate and the rainfall-runoff response of the basin. The methods by which the discharges of interest are obtained contribute to variance in the downstream hydraulic geometry parameters which is not captured in Equations (1)-(4).

To conclude, we have presented here a simple theoretical framework to describe the observed near uniformity in the spatial distribution of velocity in natural channels. Our framework is based on a conceptual basin having uniform materials throughout that exists in an equilibrium in which sediment fluxes into and out of the basin are balanced. Further, in order to describe sediment transport as a power function of channel slope and drainage area, we assumed that tectonic uplift was approximately uniform in space and constant in time. Although observed basins frequently depart from these assumptions, comparing theoretical and observed values of exponent of hydraulic radius, R, demonstrates that our framework can capture the signal of the downstream decrease in slope in the downstream adjustment of channel cross-sectional form. However, estimated values of m typically departed from observed values by a factor of approximately two, which we attribute, in part, to uncertainties in measuring and estimating the hydraulic geometry parameters and the use of a constant Manning roughness. Finally, in a departure from previous theoretical studies of hydraulic geometry, the framework we have developed assumes no underlying principles directing the evolution of channel networks toward a state which must satisfy constraints on properties such as stream power or entropy production.

Notation

A	upstream drainage area (km^2).
A_{cs}	channel cross-sectional area (m^2).
b	exponent.
b_o	exponent obtained through field inspection.
c_0	coefficient.
c_1	coefficient.
c_2	coefficient.
c_3	coefficient.
d_s	characteristic substrate size.
f	exponent.
f_o	exponent obtained through field inspection.
f_s	outgoing sediment flux.
h	flow depth.
m	exponent.
m_c	exponent computed from b_o and f_o.
m_o	exponent observed through field inspection.

n exponent.

n_0 Manning roughness coefficient.

p exponent.

P cross-sectional wetted perimeter (m).

Q discharge (m^3/s).

R cross-sectional hydraulic radius (m).

S channel bedslope.

t time.

u cross-sectional averaged velocity (m/s).

U tectonic uplift rate.

w channel flow top width (m).

z elevation (m).

z_{ss} inverse of cross-sectional side slope.

β coefficient.

γ specific weight of water-sediment mixture (N/m^3).

θ slope-area parameter.

θ_c slope-area parameter computed from b_o and f_o.

θ_o slope-area parameter obtained through field inspection.

τ average boundary shear stress (Pa).

τ_b^* bankfull dimensionless shear stress.

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Table 1: Summary of exponents of downstream hydraulic geometry equations.

Author(s)	Location	b_o	f_o	m_o	θ_o
Leopold and Maddock (1953)*	United States – gauged rivers	0.5	0.4	0.1	
Brush (1961)	Appalachians	0.55	0.36	0.09	
Langbein (1965)	Theoretical: minimum variance	0.50	0.38	0.13	-0.55
Kellerhals (1967)	Western Canada	0.50	0.40	0.10	-0.10
Emmett (1975)	Upper Salmon River, Idaho	0.56	0.34	0.12	
Rundquist (1975)	Rivers and canals with gravel and sand beds	0.52	0.32	0.16	-0.30
Li et al. (1976)	Theoretical: Threshold theory	0.46	0.46	0.08	-0.46
Charlton et al. (1978)	Britain	0.45	0.40	0.15	-0.24
Parker (1979)	Theoretical: momentum-transfer	0.50	0.42	0.08	-0.41
Chang (1980)	Theoretical: minimum stream power	0.47	0.42	0.11	
Bray (1982)	Alberta	0.53	0.33	0.14	-0.34
Andrews (1984)	Colorado: thick bank vegetation	0.48	0.37	0.14	-0.44
	Colorado: thin bank vegetation	0.48	0.38	0.14	-0.41
Hey and Thorne (1986)	Britain	0.45	0.35	0.20	-0.20
Merritt and Wohl (2003)	Yuma Wash, Arizona	0.78	0.15	0.14	

* - Authors only report one significant figure

Table 2: Values of θ_c and θ_o, for seven field studies of natural rivers.

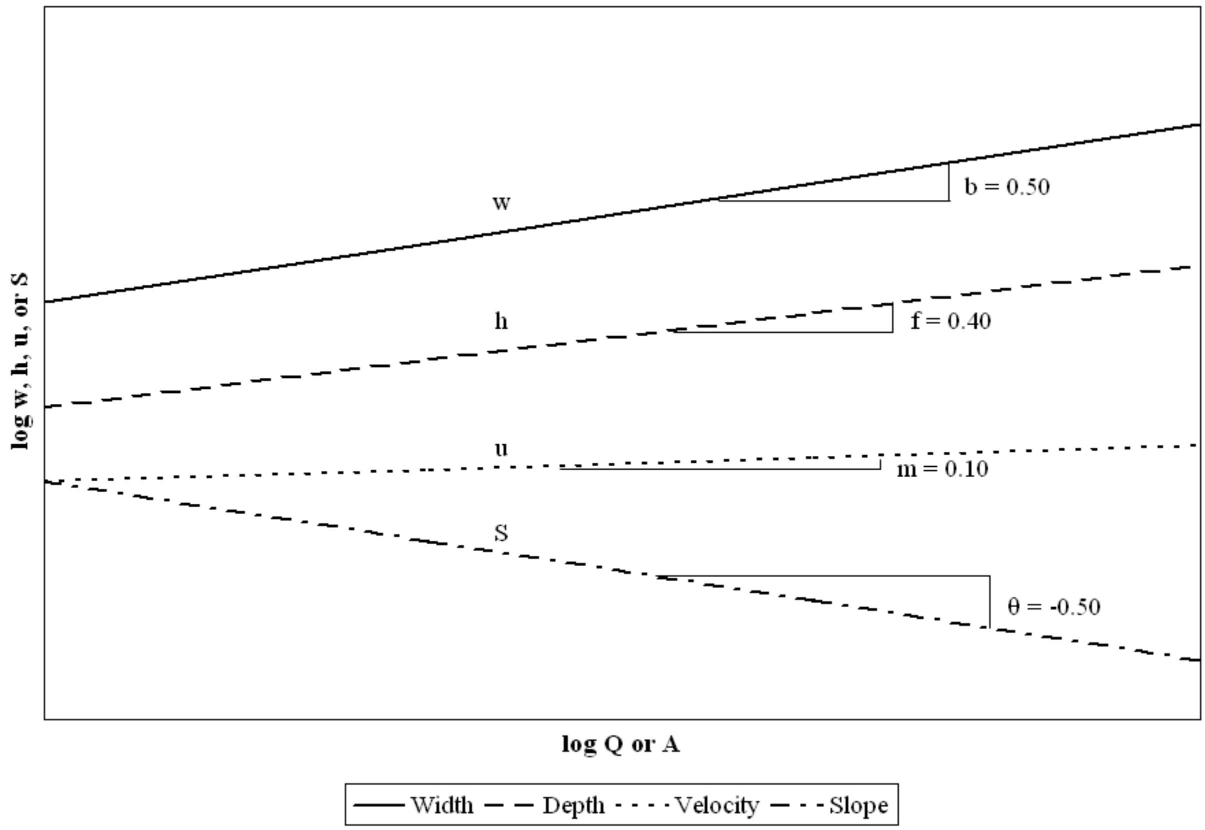
		b_o	f_o	Rectangular geometry	Triangular geometry θ_c	Trapezoidal geometry	θ_o
Kellerhals (1967)	Western Canada	0.50	0.40	-0.41	-0.40	-0.41	-0.10
Rundquist (1975)	Rivers and canals with gravel and sand beds	0.51	0.32	-0.33	-0.32	-0.33	-0.30
Charlton et al. (1978)	Britain	0.45	0.40	-0.41	-0.40	-0.40	-0.24
Bray (1982)	Alberta	0.53	0.33	-0.33	-0.33	-0.33	-0.34
Andrews (1984): thick bank veg.	Colorado	0.48	0.37	-0.38	-0.37	-0.38	-0.44
Andrews (1984): thin bank veg.	Colorado	0.48	0.38	-0.39	-0.38	-0.39	-0.41
Hey and Thorne (1986)	Britain	0.45	0.35	-0.40	-0.35	-0.36	-0.20

Table 3: Values of m_c and m_o, for seven field studies of natural rivers.

		b_o	f_o	Rectangular	Triangular m_c	Trapezoidal	m_o
Kellerhals (1967)	Western Canada	0.50	0.40	0.07	0.07	0.07	0.10
Rundquist (1975)	Rivers and canals with gravel and sand beds	0.51	0.32	0.06	0.05	0.06	0.16
Charlton et al. (1978)	Britain	0.45	0.40	0.07	0.07	0.07	0.15
Bray (1982)	Alberta	0.53	0.33	0.06	0.06	0.06	0.14
Andrews (1984) - thick bank veg	Colorado	0.48	0.37	0.06	0.06	0.06	0.14
Andrews (1984) - thin bank veg	Colorado	0.48	0.38	0.07	0.06	0.07	0.14
Hey and Thorne (1986)	Britain	0.45	0.35	0.07	0.06	0.06	0.20

FIGURE CAPTIONS

Figure 1. A schematic representation of the downstream hydraulic geometry equations. The x-axis represents discharge (Q) or basin area (A) on a logarithmic scale. Channel top width (w) is the solid black line, flow depth (h) is represented by the long dashes, cross-section averaged velocity (u) is shown with small dashes, and channel slope (S) is shown as a dash-dot line. The values of the hydraulic geometry parameters, b, f, and m are 0.5, 0.4, and 0.1 respectively following Leopold and Maddock (1953). The value of the slope-area parameter (θ) is -0.5 and represents a theoretical value for n = p = 2 in Equation (7).



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